HEAT TRANSFER DURING THE HEATING AND EVAPORATION OF A LAMINAR LIQUID FILM MOVING UNDER THE EFFECT OF A GAS STREAM

V. N. Afrosimova and E. I. Kozel'skii

UDC 532.62

A simplified physical model is examined for the flow and evaporation of a laminar liquid film entrained by a gas stream and with a given heat flux density at the wall which is constant along the entire heating surface. It is assumed that the film flow is stable, the physical properties of the liquid, except for the viscosity, depend little on the temperature, and the thermal load is not sufficient for the appearance of bubble boiling. Three characteristic sections are distinguished along the length of the film: 1) the initial thermal section within which develops the thermal boundary layer across the film thickness; 2) the section of stabilized heating of the liquid film to the equilibrium evaporation temperature; 3) the section of isothermal evaporation of liquid from the surface of the film.

Calculation of the heat transfer comes down to the joint integration of the system of equations of energy, motion, and temperature variation in the viscosity of the liquid for each of the sections and the subsequent joining of the results of the calculations. An analysis shows the possibility of simplifying the initial system of equations with allowance for the fact that: a) heat transfer in the film due to thermal conduction along the channel axis is small compared with the transfer due to convection; b) ω_y and $\partial\omega_x/\partial x$ are small compared with ω_x and $\partial\omega_x/\partial y$; c) the forces of inertia can be neglected in comparison with the forces of friction and pressure; d) the temperature dependence of the viscosity can be described accurately enough by a second-degree trinomial; e) instead of the convective term $\omega_x(\partial t/\partial x)$ in the energy equation one can be limited to its value averaged over the thickness of the film (over the the thickness of the thermal boundary layer in the first section).

In the region of stable film flow there is satisfactory agreement between the analytical solution and the results of an experimental determination of the Nusselt number obtained for film flow of hydrocarbon fuel under the effect of an air stream in a horizontally mounted evaporator channel. A certain decrease in the heat transfer compared with the calculated value, observed in the final sections of evaporation, is explained by a disturbance in the wetting of the heat-transfer surface by the film with a decrease in the irrigation density because of evaporation of part of the liquid.

Dep. 3220-74, October 21, 1974.
Original article submitted July 19, 1974.

*All-Union Institute of Scientific and Technical Information.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 28, No. 5, pp. 905-927, May, 1975.

LAWS OF HYDRODYNAMICS AND HEAT EXCHANGE DURING VAPOR FORMATION IN A FILM AT A VERTICAL SURFACE WITH WIRE INTENSIFIERS

1. STUDY OF HYDRODYNAMICS OF THE FILM

V. G. Rifert and P. A. Barabash

UDC 66.02.536.24

To intensify the heat exchange during vapor formation in a film descending over a vertical surface the authors used wires 0.7-1.47 mm in diameter fastened lengthwise to the surface. The results of a determination of some parameters of the film and the minimum irrigation density Γ_{min} are presented in the first part of the article. Analytical equations are presented for the determination of the local film thickness between wires and the average film thickness $\bar{\delta}$ over a section, its radius of curvature as a function of the distance s between wires and their diameter d, the curvature of the surface, and the wetting angle of the wires. For this one must know the thickness δ_0 at the minimum cross section of the film halfway between wires. This thickness and the local thicknesses δ at other points were determined by the electrocontact and capacitance method.

The experiments were performed on flat and cylindrical surfaces, cold and heated by electric current, made of different materials. Distilled water and an NaCl solution with a concentration of 35 g/liter served as the working liquids. At Γ_{min}, which is the flow rate below which rupture of the film occurs, the minimum values of δ_0 for surfaces with wires are three to four times lower than the minimum film thicknesses for surfaces without wires and comprise 0.03-0.05 mm for s = 4-17.5 mm regardless of the liquid temperature t_l and the overheating of the wall relative to t_l. For the determination of δ_0 for other than the minimum wetting flow rates the experimental data for any s, d, and t_l are generalized by the dependence

$$\frac{\delta_0}{\delta_0^{min}} = 1 + 14\left(\lg \frac{Re}{Re_{min}}\right)^{1.05},$$

where $Re_{min} = \Gamma_{min}/\nu\rho$ is the minimum value of Re, corresponding to the appearance of a rupture in the film upon a decrease in the liquid flow rate. The effect of the heat flux density q and of the material of the surface and the wires on Γ_{min}, s, d, and t_l was studied. Improvement in the wetting (reduction in Γ_{min}) is noted upon the appearance of vapor bubbles, as well as upon the change from distilled water to the NaCl solution. The experimental data are generalized by the equation

$$Re_{min} = 0.237\, Ga^{0.75} Z_1^{0.49} Z_2^{0.46},$$

where $Ga = g\bar{\delta}^3/\nu^2$; $Z_1 = s/\bar{\delta}$; $Z_2 = d/\bar{\delta}$, which is valid for Ga = 20-2200, Z_2 = 2-6, and $Z_1 \leq 50$.

Dep. 50-75, December 16, 1974.
Original article submitted July 7, 1974.

LAWS OF HYDRODYNAMICS AND HEAT EXCHANGE DURING VAPOR FORMATION IN A FILM AT A VERTICAL SURFACE WITH WIRE INTENSIFIERS

2. STUDY OF HEAT EXCHANGE

V. G. Rifert, P. A. Barabash, and A. A. Muzhilko

UDC 66.02.536.24

The heat-exchange mechanism is studied and the local and average heat-exchange coefficients are determined for vapor formation in a smooth film flowing down along a vertical surface with longitudinal wire ribbing. Electrical heating of the surface was used for this purpose. The local wall temperatures between the wires were measured by thermocouples, while the average wall temperatures over the sections and the average wall temperature for the entire surface were measured by thermocouples and resistors. Tests were made on distilled water and an NaCl solution with a concentration of 35 g/liter at $p = 1.17 \cdot 10^4 - 0.98 \cdot 10^5$ N/m², $q = 1.5 \cdot 10^4 - 1.2 \cdot 10^5$ W/m², and $Re_{fi} = 400-2300$. A 1.5- to 2-fold intensification of the heat exchange in the entire range of variations in q and Re_{fi} studied is noted for a surface with longitudinal wire ribbing in comparison with a smooth surface. This is explained by the presence of a thin film halfway between wires, the more turbulized film nearer the wires, and the effect on the thinner films of the pulsations arising in the thick films. A maximum of the heat exchange is obtained for a certain distance s between wires. For the case of evaporation of the film a calculating system is proposed for the determination of the average heat-exchange coefficient in which allowance is made for the presence between wires of sections differing in the modes of film flow. The experimental points lie satisfactorily near the calculated lines.

Dep. 51-75, December 16, 1974.
Original article submitted July 7, 1974.

AIR HUMIDIFICATION IN A CLOSED ROOM USING A ROTARY HUMIDIFIER

M. F. Bogomolov, V. F. Dunskii,
N. V. Nikitin, and Yu. V. Yatskov

Humidification of the air often proves to be the sole required operation in room air conditioning. In this case one can avoid bulky and expensive conditioners and be limited to the use of simple and inexpensive rotary room humidifiers. A rotating disk or drum which breaks the water down into almost identical droplets is used as the water atomizer in these humidifiers [1, 2]. Finer polydisperse satellite droplets are formed along with these "main" droplets.

The usual working process of a monodisperse atomizer (utilization of the main droplets and elimination of the satellite droplets) is reversed for rotary humidifiers; the finer rapidly evaporating satellite droplets are used for air humidification while the larger main drops are eliminated (deposited within the humidifier).

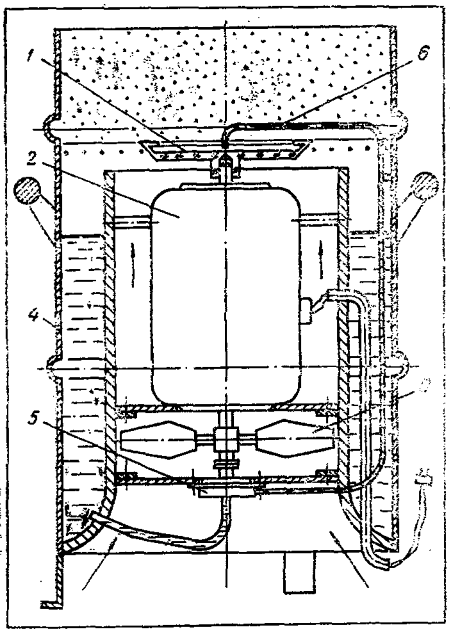

Fig. 1. Diagram of rotary humidifier.

The arrangement of a rotary humidifier [3] is shown in Fig. 1. It consists of the rotating atomizer 1 fastened to the shaft of the electric motor 2. The impeller of an axial ventilator 3 and the rotor of a centrifugal pump 5 are fastened to the lower end of the shaft of the electric motor 2. The frame 4 forms the water reservoir. When the humidifier is operating water enters from the pump 5 through the measuring nozzle 6 into the rotating atomizer 1. The main droplets thrown off from the periphery of the atomizer 1 under the effect of the centrifugal forces are deposited on the inner surface of the frame 4 and run down into the reservoir, from which the water enters the pump 5 by gravity flow. The finer satellite droplets are entrained by the air stream which is created by the ventilator 3. The turbulent air-droplet vertical jet which forms delivers the drops upward toward the ceiling and promotes the circulation and mixing of the air within the room. While settling the droplets are fully or partially evaporated and humidify the air in the room. The water flow rate is 0.17 ml/sec and the power required is 100 W.

The operation of the humidifier in a room occurs under conditions of natural ventilation. During the evaporation of the droplets they are cooled and the concentration of saturated vapor near their surface is less than the vapor concentration corresponding to saturation of the air in the room, i.e., it is not possible to provide saturation of the air in a nonhermetic room with water vapor through atomizing of water which is at room temperature. To increase the degree of saturation of the air one resorts to electrical heating of the water in the reservoir of the humidifier.

Experiments were performed in which the humidifier was placed in the center of the floor of a room with a volume of 97 m^3 and a height of 3.5 m. After 2 h of operation of the humidifier the relative humidity of the air in the room had increased from 50-60 to 75-80%.

LITERATURE CITED

1. V. F. Dunskii and N. V. Nikitin, Inzh.-Fiz. Zh., 9, No. 1, 54 (1965).
2. V. F. Dunskii, N. V. Nikitin, and N. F. Tonkacheeva, Inzh.-Fiz. Zh., 20, No. 5, 792 (1971).
3. V. F. Dunskii and N. V. Nikitin, USSR Author's Certificate No. 243, Byul. Izobr., No. 16 (1969).

Dep. 3214-74, July 25, 1974.
Original article submitted February 22, 1972.

CALCULATION OF THE THERMAL CONDUCTIVITY OF POROUS MATERIALS

V. V. Zotov, A. N. Lepilkin,
S. I. Nozdrin, and A. M. Tertychnyi

UDC 536.212

The analytical determination of the generalized conductivity coefficients of heterogeneous systems, which include porous materials, foam plastics in particular, is based on the study of models of these systems. A three-dimensional cubic lattice at the nodes of which are arranged identical pores having different shapes, most often cubical, is widely used as the model. A model with spherical pores is considered as insufficiently flexible since its porosity (Π) does not exceed 74%.

The studies which we have made, however, show that foam plastics consist mainly of closed pores, with the porosity of these materials being greater than 74%. This is explained by the presence of pores of different diameters in foam plastics. Therefore foam plastic can be considered as a material containing several porous structures, with the structures having fine pores forming the walls of the large pores.

Assuming that all the porous structures are constructed on the basis of one model — a cubic lattice with spherical pores at the nodes, and considering that the thermal conductivity coefficient of the binder (λ_1) is greater than the thermal conductivity coefficient of the gas in the pores (λ_2), equations were obtained for calculating the effective thermal conductivity of the porous structure (λ):

$$\frac{\lambda_1}{\lambda} = 1 - \left(\frac{6\Pi}{\pi}\right)^{1/3}\left[1 - \frac{a^2+1}{a}\cdot\operatorname{arc\,tg}\frac{1}{a}\right],$$

where

$$a = \left[\frac{4}{\pi\left(1 - \frac{\lambda_2}{\lambda_1}\right)\left(\frac{6\Pi}{\pi}\right)^{2/3}} - 1\right]^{1/2}.$$

By successively using the equation obtained for each porous structure, starting with the structure having the finest pores, one can calculate the thermal conductivity coefficient of a material of any porosity ($0 \leq \Pi < 100\%$). The values of the thermal conductivity coefficients of various materials calculated by the proposed method agree well with experimental values of the coefficients.

Dep. 3215-74, September 25, 1974.
Original article submitted January 9, 1974.

EXPERIMENTAL STUDY OF THE TURBULENT BOUNDARY LAYER AT A LONG ROUGH FILAMENT

V. M. Shulemovich

UDC 532.526

In the article experimental results are presented on a study of the characteristics of the boundary layer at cylinders with a diameter d = 2 mm, performed with an impinging stream velocity $u_\infty = 35$ m/sec, three heights $h_r = 2$, 35, and 85 μ of the roughness protuberances, and elongations $\lambda = 245$–1740; y and x_+ are the transverse and longitudinal coordinates.

The velocity profiles in the boundary layer were measured with a flat pneumatic probe for 10 cross sections along the filament in each experiment, corresponding to a certain height h_r. The fields of momentum loss ϑ_2 and displacement ϑ_1, the form parameter $H = \vartheta_1/\vartheta_2$, and the thickness δ of the boundary layer were determined using the experimental velocity profiles. The local friction coefficients c_f were found from the integral momentum equation.

The turbulent intensity ε of the longitudinal component of the pulsation velocity at the filament was measured in four cross sections with a constant-resistance thermoanemometer. Some results of these measurements are presented in Fig. 1.

It is shown experimentally that the roughness function Φ in explicit form does not depend on the transverse curvature.

The effect of the curvature was manifested, for example, in the fact that the corresponding values of the average (c_F) and local friction coefficients at the cylinder are the higher compared with those for a plate, the greater the height of the roughness protuberances and the elongation of the cylinder. The thickness of the boundary layer and the form parameter are lower at a cylinder than at a plate.

The maximum differences obtained in these experiments for c_F, δ, and H at a rough cylinder and at a plate are about 100, 50, and 14%, respectively.

The experimental data are in satisfactory agreement with the results of theoretical calculations.

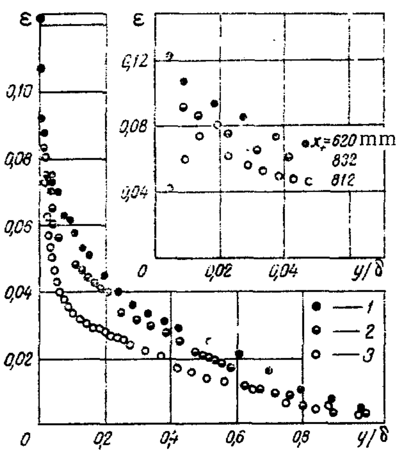

Fig. 1. Turbulent intensity at the filament: 1) $h_r = 0.085$ mm; 2) 0.035; 3) 0; for 1-3) $u_\infty = 35$ m/sec.

Dep. 3218-74, October 28, 1974.
Institute of Theoretical and Applied Mechanics, Siberian Branch Academy of Sciences of the USSR, Novosibirsk.
Original article submitted May 25, 1974.

EXPERIMENTAL INVESTIGATION OF BUBBLE-FORMING ACTION OF PORES

I. I. Markov

UDC 536.423.1

The bubble-forming action of pores in the boiling of a liquid was experimentally investigated. The following aspects of this question were considered.

1. How does the solution of air from pore cavities take place in open and closed vessels?

2. How does residual gas in pores affect their activity, and how does the temperature head ΔT, which determines the onset of bubble formation on the isolated potential boiling center, depend on the pore diameter?

3. What effect does mechanical action have on the bubble-forming action of pores?

It can be concluded from the obtained experimental data that gas from pore cavities dissolves in a closed vessel at a lower rate than in an open vessel. Pores can act much longer as potential boiling centers in a closed vessel than in an open vessel. The gas solution rate increases in open and closed vessels with increase in temperature of the liquid from room temperature to saturation temperature T_S. The smaller the diameter of the pore (capillary) outlet, the greater the rate of gas solution. Pores which have lost all their gas become completely filled with liquid at liquid temperature $T < T_S$ and cease to be active boiling centers.

A reduction of the partial pressure of gas in a pore leads to an increase in the temperature head ΔT at which bubble formation begins. In the uniform field of a copper heater the temperature head increases linearly with reduction in pore diameter. On a glass heating surface, beginning at a pore diameter of 0.1 mm, there is a sharp increase in temperature head with reduction in diameter of the pore (capillary). Since the air solution rate increases with reduction in pore diameter it can be inferred that on pores with diameter $d < 0.1$ mm the partial pressure of gas at the start of boiling is practically zero, which leads to considerable overheating.

In the nonuniform field of a heater an increase in pore depth leads to a reduction of the active pore radius.

Vibration of the heating surface in a vertical direction with frequency $0 < f \leq 200$ Hz and amplitude $0.2 \leq A \leq 1.25$ mm does not alter the number of vapor-forming centers. On functioning vapor-forming centers an increase in the frequency of the vibrating heating surface reduces the bubble breakaway diameter D_0, which in this case can be calculated from the formula:

$$D_0 = \sqrt[3]{8cR/g},$$

where c is a constant which depends on the kind of liquid; R is the pore radius; g is the acceleration. The frequency f_0 of bubble breakaway increases; the boiling rate is $f_0 D_0$.

Mechanical action at sites of breakaway of liquid from the heating surface leads to impact boiling of the liquid. If a break occurs in a passive pore (capillary), this site becomes a stable boiling center.

Dep. 52-75, October 30, 1974.
Original article submitted September 17, 1974.

RELATION BETWEEN ACOUSTIC SIGNAL AND OSCILLATIONS OF A VAPOR FILM DURING UNDERHEATED FILM BOILING

V. V. Chekanov and L. G. Berro

UDC 534.8:536.423.1

It has been noted [1-3] that in the case of a pronounced underheating the film boiling which occurs at a thin wire is accompanied by the generation of sound — a whistle at a pure tone. Our purpose in the

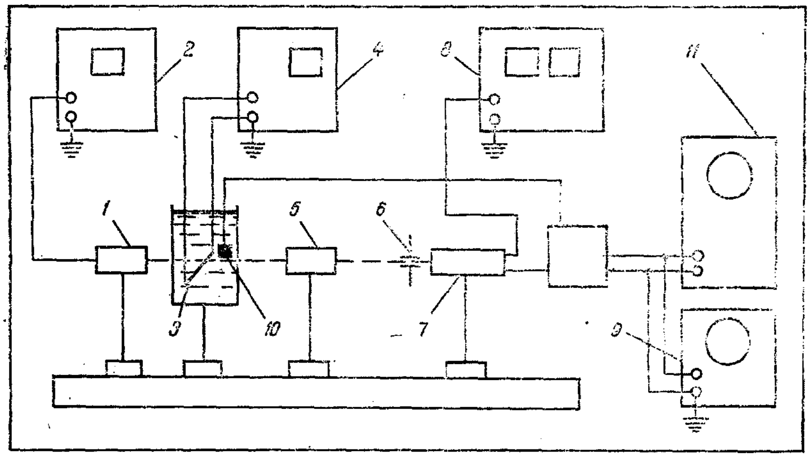

Fig. 1. Experimental apparatus. 1) Light source; 2, 4) stabilized dc power supplies; 3) heater; 5) objective lens; 6) slit; 7) photomultiplier; 8) all-purpose power supply; 9) oscilloscope; 10) hydrophone; 11) spectrum analyzer.

present study was to detect the oscillations of the vapor-filled cavity which appears around the heater, to determine the frequency of these oscillations, and to determine the relationship between these oscillations and the amplitude — frequency spectrum of the sound emitted under these conditions.

Figure 1 shows the experimental apparatus; the experimental procedure is described in [4]. The apparatus is assembled around an OSK-2 optical bench. The processes which occur at the surface of the heater 3 are observed in transmitted light, so that a clearly defined shadow of the vapor-filled cavity is found on the cathode of photomultiplier 7. The oscillations of the cavity modulate the light flux, and the electrical signal from the photomultiplier, which is fed to oscilloscope 9, is proportional to the displacement of the vapor film.

The noise which occurs upon boiling is detected by hydrophone 10 and studied by the SK4-3 noise spectrum analyzer 11. The experiments are carried out under conditions such that the underheating of the liquid is

$$\Delta T_{\text{und}} = T_{\text{sat}} - T_{\text{l}} \sim 35 - 60°.$$

The boiling occurs on thin tungsten wires 0.12 and 0.3 mm in diameter, immersed in alcohol (ethyl or propyl). The experiments are carried out in vessels of various sizes and shapes. The use of a dual-trace oscilloscope makes it possible to determine the phase relations of the signals from the hydrophone and from the photomultiplier.

Analyzing the data, we can draw the following conclusions.

1. The emission of a pure acoustic tone under these conditions results from oscillations of the volume of the vapor-filled cavity around the heater.

2. In the case of underheated film boiling, the frequency of the principal maximum in the spectrum of the acoustic signal is equal to the frequency of the volume oscillations of this cavity.

3. As the heat flux is increased, the frequencies of the principal maxima of both signals decrease.

LITERATURE CITED

1. E. I. Nesis, Boiling of Liquids [in Russian], Nauka, Moscow (1973).
2. M. F. M. Osborne, J. Acoust. Soc. Amer., 19, No. 1, 21 (1947).
3. B. M. Dorofeev, L. G. Berro, V. A. Assman, Yu. I. Dikanskii, and N. S. Sergeev, in: Research on the Physics of Boiling [in Russian], No. 2, Stavropol' (1974).
4. V. V. Chekanov, in: Research on the Physics of Boiling [in Russian], No. 2, Stavropol' (1974).

Dep. 60-75, September 12, 1974.
Original article submitted April 25, 1974.

FORCED COOLING OF CONSTRUCTIONAL ELEMENTS IN CRYOGENIC DEVICES

A. V. Zhukovskii

UDC 536.24

It is common practice to reduce the heat leakage to the cold zone in a cryogenic device by cooling the constructional elements by means of the escaping gas; the heat leaks and temperature distributions in such elements may be derived from an analytical solution for the one-dimensional thermal conduction in a rod of constant cross section, part of which is cooled by a flow of cold gas towards the warm end of the rod. The thermal conductivity of the rod is assumed constant and equal to the integral mean value for the cooled part λ and uncooled part λ_L. The following quantities are specified: the cross section of the rod F, cooled length l, and uncooled length L; cooled perimeter of the rod U, gas flow rate G, gas temperature at the inlet to the cooling section ϑ_l, and temperatures at the ends of the rod: hot end T_0 and cold end T_l, as well as the heat-transfer coefficient to the gas α, and the mean specific heat of the gas c_p.

Expressions for the temperature distribution in the rod, the temperature of the coolant, and the heat leak to the cold end are derived in dimensionless form; the solution contains not only the boundary conditions for T_l/T_0 and ϑ_l/T_0, but also the three quantities $Gc_pl/\lambda F$, $\alpha Ul/Gc_p$ and Gc_pL/λ_LF (or combinations of these). If the entire rod is cooled by the gas, it is found that the heat leak to the cold end as a ratio of the heat leak without cooling, Q_l/Q_{max}, is governed solely by $Gc_pl/\lambda F$ and $\alpha Ul/Gc_p$, where $Q_{max} = \lambda F/l(T_0 - T_l)$, and this relationship is shown in Fig. 1, which allows one to deduce the heat leak Q_l along the rod.

Experiments indicate that the calculations give somewhat overestimates for the heat leads, since the solution does not incorporate the variation in the thermal conductivity of the rod and the heat-transfer coefficient along the length. Detailed calculations have been made for particular rod materials and cooling agents on the basis of the temperature dependence of the properties, using numerical integration; the general trends shown in the figure still persist.

These simple solutions can be valuable in estimating heat leaks and temperature distributions in gas-cooled structural elements of cryogenic equipments, and also for defining the gas flow needed (with a certain safety factor) to produce a given reduction in the heat leak to the cold zone.

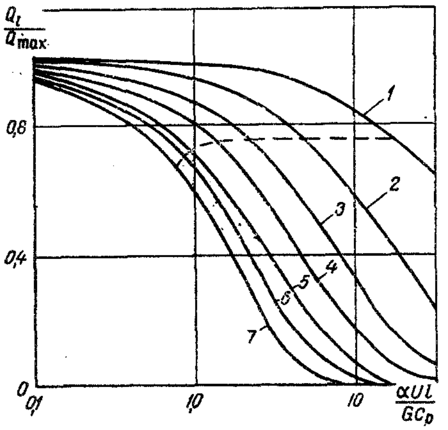

Fig. 1. Relative heat flux from cooled rod as a function of dimensionless quantities. Values of $\alpha Ul/Gc_p$: 1) 0.1; 2) 0.4; 3) 1.0; 4) 2.0; 5) 5.0; 6) 10; 7) ∞. Above the broken line the difference is less than 10% from the results with variable thermal conductivity for a stainless-steel rod cooled by helium at $20K \leq T \leq 300K$.

Dep. 59-75, December 13, 1974.
Original article submitted December 10, 1973.

PROCESSES IN HEAT BRIDGE FLUSHED VIA A MEDIUM AGAINST THE HEAT FLUX

L. M. Rozenfel'd and A. G. Korol'kov

UDC 621.1.016.4:621.59+536.24.242

Heat bridges occur in cryostats and other such devices and are sources of unwanted heat leak, which can be reduced by cooling the bridges with the cold gas flowing against the heat flux [1].

Consider a bridge between cold and warm media cooled as shown in Fig. 1; the conditions are static, with given temperatures for the media and given heat-transfer coefficients at the ends and middle.

It is assumed that the physical properties of the bridge and gas are constant, and the longitudinal thermal conduction in the gas is neglected, which gives a solution to the thermal-conduction equations for the bridge and the energy equation for the gas in terms of the dimensionless temperatures of bridge and gas averaged over the cross section, the values being expressed in terms of the Stanton and Biot numbers, together with the geometrical similarity of the cooled part, in conjunction with dimensionless quantities that represent the heat transfer at the ends of the bridge, which have been given in [2].

The characteristics of such a bridge are presented, in particular, as heat fluxes at the hot and cold end as ratios to the heat flux without cooling, as well as the gas temperature as a ratio to the temperature difference between the media, again in terms of the dimensionless variables.

An analysis is presented for the effects on the characteristics from the dimensional parameters of the bridge and gas over a wide range.

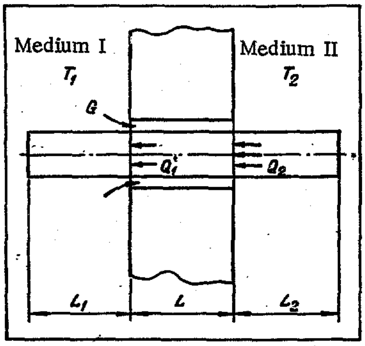

Fig. 1. Model for calculations on a cooled heat bridge. T_1 and T_2 are the temperatures of the cold and warm media, G is the cooling-gas flow rate, and Q_1 and Q_2 are the heat fluxes, respectively, into the cooled end and into the cooled bridge.

LITERATURE CITED

1. L. M. Rozenfel'd and N. N. Koshkin, "Dynamic thermal insulation," Zh. Tech. Fiz., 24, No. 1 (1954).
2. S. S. Kutateladze, Principles of Heat-Transfer Theory [in Russian], Sibirsk. Otd., Nauka, Novosibirsk (1970).

Dep. 58-75, November 5, 1974.
Original article submitted October 17, 1973.

PROPERTIES OF THE RESISTANCE OF ROTATING POROUS BODIES

S. I. Kulikovskii

UDC 66.071.6

The physical processes accompanying the flow of a liquid or gas through a cylindrical, porous, rotating body are analyzed in the article. Models of the serial type are used to describe the nature of the flow. Using an equation of flow with allowance for inertial losses with slippage, an expression is obtained for the pressure distribution both in a stationary and in a rotating porous rotor. Values of the permeability coefficients are given for molecular and viscous modes of flow. Two means of gas supply are used: along and against the effect of the centrifugal field. The data of the experimental studies are compared with computer calculations.

With gas flow along the effect of the field and a low rotation speed the distribution hardly differs from the distribution in a stationary diffuser. With an increase in the speed the pressure increases in the peripheral regions and decreases in the middle part of the rotor. The smaller the permeability, the smaller the distance from the axis at which this increase begins to appear. The reason for this effect is the nonlinear relationship between the stress tensor and the tensor of the deformation rate.

With movement of the gas against the effect of the field the resistance of the diffuser increases with an increase in the rotor speed. This is due to the joint effect of two forces — the resistance of the porous material and the centrifugal field.

Dep. 3217-74, September 27, 1974.
Original article submitted March 29, 1974.

ISOTHERMAL MOVEMENT OF VAPOR IN A TUBE

Ya. A. Levin, É. B. Filippov, and A. A. Yarkho

UDC 532.517.4

The established isothermal turbulent flow of a vapor along a round cylindrical tube is examined in a one-dimensional formulation to find the dependence of the pressure losses in the pipelines of a heat-exchange apparatus on the flow rate of dry vapor with allowance for the real properties of the latter. The equation in virial form, in which terms containing ρ to the second power or higher were neglected, was used as the equation of state of a real gas. The equation of state was solved for ρ. Of the two roots of the equation of state obtained, the smaller, satisfying the region of superheated vapor with a temperature below the critical temperature, was taken.

It is known that up to Mach numbers $M \approx 0.8$ the coefficient of resistance ξ is a function only of the Reynolds number Re. For the movement of a gas along a round cylindrical tube, Re is, in turn, a function of the viscosity coefficient η, which depends only on the temperature when the pressure variations are not very large. Consequently, with isothermal gas flow Re remains constant along the pipe, which means the coefficient of resistance ξ will also be constant.

With these conditions the system of equations of continuity, motion, and of state can be integrated. After several elementary transformations the expression for a long pipeline obtained as a result of the integration will have the form

$$\frac{B_1}{6RT}(p_0^3-p^3)+\frac{1}{2}(p_0^2-p^2)=\frac{G^2}{F^2}RT\xi\frac{l}{2D}, \tag{1}$$

where p is the pressure; T is the temperature; R is the gas constant; B_1 is virial coefficient of the equation of state; G is the mass flow rate; F is the cross-sectional area of the pipeline; l is the length of the pipeline; D is the inner diameter of the pipeline; ξ is the coefficient of friction.

Equation (1) represents a general expression connecting, with the assumptions indicated above, the parameters of a moving isothermally real gas. From it one can obtain the well-known expressions for the determination of the pressure losses during the movement along a pipeline of an incompressible liquid (the Darcy — Weisbach equation) and of an ideal gas.

Equation (1) can be used to determine the mass flow rate of a vapor moving isothermally along a pipeline with known geometrical characteristics when the pressure drop is given. If $Re < 10^5$ then the Blasius equation can be taken for the coefficient of friction. Then after transformations of Eq. (1) we obtain

$$G^{1.75}=2{,}07\frac{D^{4.75}}{lRT\eta^{0.25}}\left[p_0^2-p^2+\frac{B_1}{3RT}(p_0^3-p^3)\right]. \tag{2}$$

A comparison of calculations by Eq. (2) and experiments with nitrogen and helium showed that the error in the determination of the mass flow rate of a vapor moving isothermally along a cylindrical pipeline does not exceed 7%. Not allowing for the true properties of the vapor can lead to an error of 20-30%.

Dep. 57-75, November 18, 1974.
Original article submitted July 11, 1974.

AERODYNAMIC CHARACTERISTICS OF SLOTTED DRAINS

E. P. Vishnevskii

UDC 532.542

Slotted drains are analyzed, with the area F(x) of the air collecter and width $\varepsilon(x)$ of a slot being functions of the distance from the dead end.

In this case the velocity distribution along the axis of the air collecter is

$$w^* = \frac{w}{w_0} = \frac{\operatorname{sh} \mu\Phi (x^*, I_1, I_2, \ldots,)}{\operatorname{sh} \mu I_1},$$

where x* is the dimensionless coordinate; I_j are geometrical invariants (dimensionless complexes of the structural parameters of the device); μ is the flow-rate coefficient.

The velocity in a slot is

$$v^* = \frac{v}{v_0} = \frac{\operatorname{ch} \mu\Phi (x^*, I_1, I_2, \ldots,)}{\operatorname{ch} \mu I_1}.$$

The coefficient of aerodynamic resistance is

$$\zeta = \operatorname{cth}^2 \mu I_1 - 1.$$

The variation of the velocity distribution in a slot is

$$\eta = \left(1 - \frac{1}{\operatorname{ch} \mu I_1}\right) \cdot 100\,\%.$$

Four particular cases are examined for which calculating equations are derived. The specific content of the invariants I_j in this case is determined by the geometry of the devices in accordance with the equations

$$I_1 = \int_0^l \frac{\varepsilon (x)}{F (x)}\, dx,$$

$$\Phi (x^*, I_1, I_2, \ldots,) = \int_0^x \frac{\varepsilon (x)}{F (x)}\, dx.$$

Dep. 3256-74, October 18, 1974.
Original article submitted August 23, 1973.

HYDRODYNAMICS OF TURBULATORS FOR THE TWISTING OF THE AIR STREAM AT THE ENTRANCE TO A TUBE

A. M. Voitko

UDC 532.5

A preliminary analysis of the energy expenditures in overcoming the resistance of tubular heat-exchangers having a turbulent air stream shows that a considerable part of the pressure head is expended in overcoming the resistance of the turbulators, in connection with which it is desirable to determine the effect of their structural properties on the coefficient of resistance. The investigation was concerned with three designs of turbulator: with a smooth tangential connection of air flow formed by half-circles with

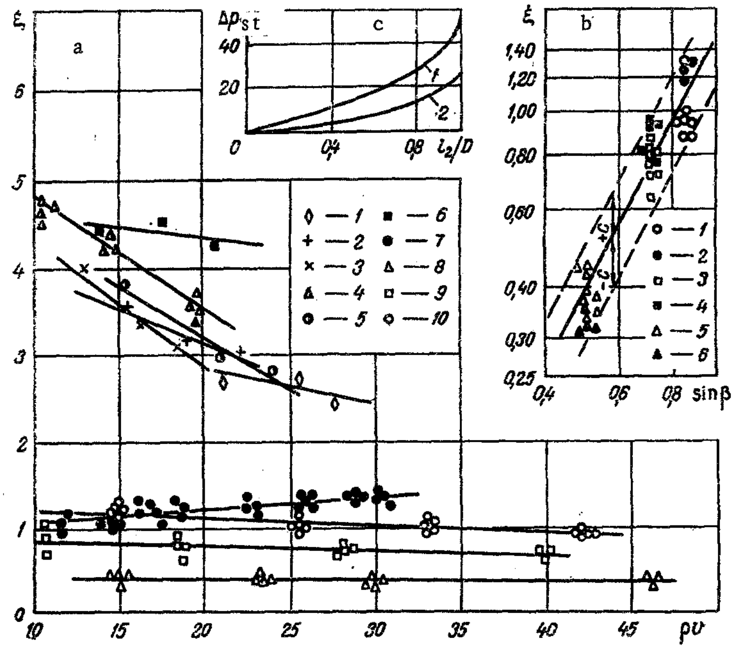

Fig. 1. Hydrodynamic characteristics of turbulators: a) $\xi = (\rho v_t)$, turbulator with tangential airflow a × b = 140 × 140 mm (1), 160×160 (2), 180×180 (3), 400×400 (4), 130×200 (5), and 213×130 mm (6)); turbulator with flat vanes (β = 60°, D = 406 mm (7)); turbulator with smooth bending of vanes (β = 30° (8), 45° (9), 60° (10), D = 220 mm); b) swirler with smooth bending of vanes, $\xi = f(\sin\beta)$ valid for $\rho v_t \approx 5$-50 kg/m² · sec, 1, 3, 5) β = 60, 45, and 30°, respectively, l_1 = D = 220 mm, 2, 4, 6) β = 60, 45, and 30°, respectively, l_1 = D = 406 mm, c = ±25%; c) dependence of Δp (mm H_2O) on l_2/D and ρv_t [ρv_t = 39 (1) and 26.7 kg/m² · sec (2)].

centers located at equal distances from the tube axis and with an open or closed diaphragm (tube diameter D = 200 mm, diaphragm diameter d = 100 mm with the inlet cross section a × b, where a is the channel length and b is the channel width), a register with eight flat vanes mounted at an angle β = 60° to the generatrix of the tube (tube diameter D = 406 mm), and a turbulator also with eight turbulating vanes but with the angle β varying smoothly from 0° (at the entrance of the air stream to the turbulator) to 30, 45, and 60° (at the exit of the air stream from the turbulator with a tube diameter D = 220 mm). For the second and third turbulator the length was equal to their diameter.

The studies of the coefficient of resistance $\xi = f(\rho v_t)$ of the turbulators from the mass velocity in a very narrow cross section of the channel were preceded by studies of the velocity fields (tangential and axial) of the turbulent stream in the initial cross section of the tube (l/D = 1.5 for the first turbulator and l/D 3.7 for the third turbulator, l is the running length of the tube), as a result of which it was established that their difference is insignificant (with β = 60°) whereas their coefficients of resistance differ considerably from one another (Fig. 1a).

The latter, which is energetically advantageous, distinguishes the third turbulator from the first, in connection with which it was studied in more detail. The variation of the difference in static pressures between the entrance cross section of the turbulator), (β = 60°) and along the length of its channels (l_2 is the current length of the channel) was checked for different values of ρv_t (Fig. 1c). It is seen from Fig. 1a that the coefficient of resistance decreases considerably with a decrease in the angle β (third swirler), although reorganization of the velocity fields occurs in this case with a shift in the high-velocity zones from the wall of the tube toward the axis, which reduces the intensity of heat exchange between the air stream and the wall of the tube. Therefor the choice of a turbulator in each concrete case must be justified by a technical — economical calculation.

Dep. 55-75, November 6, 1974.
Original article submitted April 2, 1973.

NUMERICAL SOLUTION OF THE HEAT EQUATION WITH A CONVECTIVE TERM

N. V. Astanovskaya and A. A. Pirozhenko

UDC 518.5:517.946

We consider the problem of the propagation of heat in a moving, chemically inert medium for the condition that energy sources are absent and the energy dissipation is negligibly small. The boundary conditions for the heat-propagation equation

$$\frac{\partial(\rho C_p T)}{\partial t} + \operatorname{div}(\bar{u}\rho C_p T - \lambda \operatorname{grad} T) = 0,$$
$$x_\sigma \in G, \quad \sigma = 1, 2, \tag{1}$$

with account of the symmetry of the heating conditions and the geometry of the region being considered are:

1) the symmetry conditions

$$\frac{\partial T(x_1, 0, t)}{\partial x_2} = 0, \quad l_w \leqslant x_1 < l_1, \quad t > 0; \tag{2}$$

$$\frac{\partial T(0, x_2, t)}{\partial x_1} = 0, \quad 0 \leqslant x_2 < l_2, \quad t > 0; \tag{3}$$

2) the given temperature distributions

$$T(x_1, x_2, t) = T_2, \quad (x_1, x_2) \in S \text{ — the external surface} \tag{4}$$

$$T(x_1, x_2, t) = T_1, \quad 0 \leqslant x_1 \leqslant l_w \text{ — the surface of the base} \tag{5}$$

For a numerical solution of Eq. (1) with the boundary conditions (2)-(5) and the initial condition

$$T(x_1, x_2, 0) = T^0, \quad (x_1, x_2) \in G$$

we use an absolutely stable scheme, constructed on the basis of a local one-dimensional method [1], which, for the chosen order of approximation, converges with rate $\sim o(|h| + \tau)$.

The velocity distribution of the gas phase is given, and is an approximation of the motions of the gas phase determined in [2] for each interval of critical Rayleigh numbers $[Ra_{i-1}^{cr}, Ra_i^{cr}]$.

In the study we carry out a theoretical and numerical investigation of the stability and the convergence of the proposed scheme, and we represent results of calculations of the temperature fields for channels with various configurations of the cross section.

We note that the presence of free convection can considerably influence the nature of the heat transfer on the base and the behavior of the local Nusselt number

$$\mathrm{Nu}(x) = \frac{\lambda_* \left.\frac{\partial T}{\partial x_2}\right|_0^{l_2}}{\lambda (T_2 - T_*)},$$

where T_* is the average temperature over the volume.

LITERATURE CITED

1. A. A. Samarskii, Introduction to the Theory of Difference Schemes [in Russian], Nauka, Moscow (1971).
2. G. Z. Gershuni and E. M. Zhukovitskii, Convective Stability of an Incompressible Liquid [in Russian], Nauka, Moscow (1972).

Dep. 61-75, November 15, 1974.
NIPTI, Tallin.
Original article submitted December 27, 1973.

OPTIMAL SYMMETRIC TEMPERATURE REGIME FOR A PLATE

V. M. Vigak and A. A. Fedorishin

UDC 536.21:539.377

The problem of the control of start-up processes in thermal power equipment is connected with the problem of determining the optimal unsteady temperature regime for separate elements, ensuring over the entire extent of the transient process, given limiting admissible temperature stresses at the maximally stressed points of the element.

To determine the optimal unsteady temperature regime in an unbounded plate of thickness 2h we consider the solution of the heat equation, for which as boundary conditions the condition of symmetry of the temperature field over the thickness of the plate is chosen, and the condition of the equality of the temperature stresses on the outer surfaces is admissible, which is written in the form

$$\frac{1}{2}\int_{-1}^{1} T(\rho,\ \mathrm{Fo})\, d\rho - T(1,\ \mathrm{Fo}) = b - cT(1,\ \mathrm{Fo}). \tag{1}$$

Here the left side of condition (1) is an expression for the calculated relative temperature stresses on the surface of the plate $\rho = 1$, and the right side is an expression for the admissible relative temperature stresses $S = [(1-\nu)/\alpha_T E]\sigma$, the variation law of which depending on the temperature is assumed to be approximated by a broken line; in separate temperature ranges these stresses are represented by the linear function $S = b - cT(1,\ \mathrm{Fo})$.

The solution of this heat-conduction problem is obtained in the form

$$T(\rho,\ \mathrm{Fo}) = \frac{b}{c} + A_1 \operatorname{ch} \lambda_1\rho \exp(\lambda_1^2 \mathrm{Fo}) + \sum_{n=2}^{\infty} A_n \cos \lambda_n\rho \exp(-\lambda_n^2 \mathrm{Fo}), \tag{2}$$

where λ_1 and λ_n ($n = 2, 3, \ldots,$) are the roots of the corresponding characteristic equations; the coefficients A_n ($n = 1, 2, \ldots,$) are determined from an infinite system of algebraic equations, obtained on the basis of the initial condition.

It is shown that unlike the heat-conduction problem in the usual formulation, the optimal problem with a constraint based on the stresses, besides solution (2), has a particular solution for $\lambda_1 = 0$.

In a similar manner we find the principal and particular solutions of the optimal heat-conduction problem for a constraint on the temperature stresses in the middle plane of the plate.

The optimal temperature regime that is found allows us, for example, from the boundary condition of second or third kind on the outer surface of the plate, to determine a control function ensuring given admissible temperature stresses, or a thermal flux, or the temperature of the heating medium for the corresponding Biot criterion, or, on the contrary, the Biot criteria for given temperature of the heating medium.

The solution obtained for the optimal unsteady heat-conduction problem can be used for determining the control function needed for automatic control of the start-up regimes of thermal power equipment.

NOTATION

$\rho = x/h$, dimensionless coordinate; x, axis of coordinates with origin on the middle plane of the plate; $\mathrm{Fo} = a\tau/h^2$, Fourier criterion; a, thermal-conductivity coefficient; b, c, constant coefficients determined from the given straight line of admissible relative temperature stresses S; ν, α_T, the Poisson ratio and the linear-expansion coefficient; E, elastic modulus.

Dep. 54-75, November 22, 1974.
Original article submitted January 9, 1974.

SOLUTION OF A THREE-DIMENSIONAL HEAT-CONDUCTION PROBLEM FOR AN INHOMOGENEOUS ORTHOTROPIC SEMI-INFINITE HOLLOW CYLINDER FOR MIXED BOUNDARY CONDITIONS

P. Z. Livshits

UDC 536.2

Assuming a power dependence for the thermal-conductivity coefficient on the radial coordinate ρ we construct solutions for the equation of steady heat conduction for an unbounded hollow cylinder corresponding to homogeneous mixed boundary conditions

$$\rho = 1+\lambda : T_\nu = 0,\ \ \xi \geqslant 0;\ \ \frac{1}{h_{ra}R}\cdot\frac{\partial T_\nu}{\partial \rho} + T_\nu = 0,\ \ \xi < 0,$$

$$\rho = 1-\lambda : \frac{1}{h_{rb}R}\cdot\frac{\partial T_\nu}{\partial \rho} - T_\nu = 0,\ \ -\infty < \xi < \infty.$$

Using the solutions that have been constructed we consider two problems on the arbitrary heating of the face of a semibounded cylinder for homogeneous mixed boundary conditions ($l = c/R$):

$$\rho = 1+\lambda : T_\nu = 0,\ \ 0 \leqslant \xi \leqslant l;\ \ \frac{1}{h_{ra}R}\cdot\frac{\partial T_\nu}{\partial \rho} + T_\nu = 0,\ \ -\infty < \xi < 0;$$

$$\rho = 1-\lambda : \frac{1}{h_{rb}R}\cdot\frac{\partial T_\nu}{\partial \rho} - T_\nu = 0,\ \ -\infty < \xi \leqslant l; \tag{1}$$

$$\xi = l : T_\nu = f(\rho),\ \ 1-\lambda \leqslant \rho \leqslant 1+\lambda;$$

$$\rho = 1+\lambda : T_\nu = 0,\ \ 0 \leqslant \xi < \infty;\ \ \frac{1}{h_{ra}R}\cdot\frac{\partial T_\nu}{\partial \rho} + T_\nu = 0,\ \ -l \leqslant \xi < 0;$$

$$\rho = 1-\lambda : \frac{1}{h_{rb}R}\cdot\frac{\partial T_\nu}{\partial \rho} - T_\nu = 0,\ \ -l \leqslant \xi < \infty; \tag{2}$$

$$\xi = -l : T_\nu = f(\rho),\ \ 1-\lambda \leqslant \rho \leqslant 1+\lambda.$$

We solve the problem of the uniform (in the axial direction) heating of a section of length 2c of the external lateral surface of an unbounded hollow cylinder for heat exchange with a medium at zero temperature on the remaining part of the external surface and on the entire internal surface.

We find a solution for the problem of an unbounded cylinder heated over the entire external lateral surface, except for a finite (2c) section of heat exchange.

In all the problems considered in the article the conditions on the lateral surfaces are satisfied exactly. The coefficients in series based on homogeneous solutions are determined from the normal (according to Poincare and Koch) system of algebraic equations.

We obtain asymptotic equations which enable us to find the intensity of the radial thermal flux on the external lateral surface near the separation line of the boundary conditions.

We carry out a conversion to cases of modified boundary conditions — the thermal insulation of the part $\xi < 0$ of the external surface (i.e., $h_{ra} \to 0$). We consider in detail the axisymmetric problem with modified boundary conditions in the particular case $m = -1$. Results of the calculations make it possible for us to judge the effect of the orthotropicity of the material of the cylinder and the finiteness of the length of the heating section for various relative thicknesses b/a of the cylinders.

NOTATION

$\rho = r/R$, $\xi = z/R$, dimensionless coordinates [a) external, b) internal, $R = (a+b)/2$) mean radii of the cylinder]; $\lambda = a - b)/2R$, half of the relative thickness of the cylinder wall; h_{ra}, h_{rb}, relative coefficients of the heat transfer on the external and internal surfaces; $T_\nu(\rho,\ \xi)\cos\sqrt{\nu^2 - (m/2)^2/g_2}\ \varphi$, particular solution of the heat-conduction equation $\partial/\partial\rho\,(\rho\lambda_r(\partial T/\partial\rho)) + \partial/\partial\varphi(\lambda_\varphi/\rho\cdot\partial T/\partial\varphi) + \partial/\partial\xi\,(\rho\lambda_z(\partial T/\partial\xi)) = 0$; $g_2 = \lambda_\varphi/\lambda_r$; $\nu^2 - (m/2)^2 \geq 0$; $\lambda_i = \lambda_i^0\rho^m$, thermal-conductivity coefficients ($\lambda_i^0 = \text{const}$, $i = r,\ \varphi,\ z$).

LITERATURE CITED

1. A. A. Lykov, Theory of Heat Conduction [in Russian], Vysshaya Shkola, Moscow (1967).
2. B. M. Nuller, Prikl. Mat. Mekh., 34, No. 4, 621-631 (1970).
3. B. Noble, Methods Based on the Wiener – Hopf Technique for the Solution of Partial Differential Equations, Pergamon (1958).
4. P. Z. Livshits, Prikl. Mekh., 8, No. 11, 9-14 (1972).
5. P. Z. Livshits, Inzh.-Fiz. Zh., 24, No. 6 (1973).

Dep. 63-75, December 10, 1974.
Institute of Pharmaceutical Chemistry, Leningrad.
Original article submitted December 10, 1973.

USE OF ORDINARY DIFFERENTIAL EQUATIONS TO DESCRIBE THE HEATING OF ELEMENTS OF ARBITRARY SHAPE

A. Kh. Gorelik

UDC 536.24.02

Lately much consideration has been given to a description of the heating of turbine apparatus, for example, in order to construct programs for the increase of the parameters of the vapor or gas during start-up based on given limiting differences or velocities of temperatures at various points of the elements (see, e.g., [1]).

In [2] ordinary differential equations were obtained that connected the temperature of the metal at separate points of a cylinder and a plane wall with the temperature of the heating medium, which varied in time.

In the present article the results obtained in [2] are generalized for the prediction of the heating of elements of arbitrary shape. The structure of the equations of heating are determined, and the relations between the coefficients of the equations, the shape of the elements being heated, and the initial conditions are shown.

For heating (cooling) of bodies by a heating medium having the same temperature on the section supplying the heat, with insulation from the unheated section of the surface or with loss of heat on this section it is shown that the temperature at a point (coordinates of the point x, y, z) of the body being heated $\Theta(x, y, z, t)$ with variation of temperature of the heating medium $\Theta_0(t)$ in time t can be represented as a sum of the reactions of an infinite number of aperiodic links with coefficients of the force $M_n(x, y, z)$ that decrease as a function of the number, and with time constants T_n, and also the initial damping component:

$$\Theta(x, y, z, s) = \Theta_0(s) \sum_{n=1}^{n=\infty} \frac{M_n(x, y, z)}{T_n s + 1} + s \sum_{n=1}^{n=\infty} \frac{N_n(x, y, z) T_n}{T_n s + 1},$$

where s is an operator; $M_n(x, y, z)$ are coefficients that depend on the shape of the body being heated and the coordinates of the point; T_n are coefficients that depend on the shape and conditions of heat exchange on the surfaces of the body being heated; $N_n(x, y, z)$ are coefficients that depend on the initial temperature distribution, the coordinates of the point, and the shape of the body being heated.

In accordance with this expression, the temperature at any point of the body being heated can be determined as the solution of the following ordinary differential equation with right side

$$\left[\prod_{n=1}^{n=\infty} (T_n p + 1)\right] \Theta(x, y, z, t) = \left[\sum_{m=1}^{m=\infty} M_m(x, y, z) \prod_{\substack{n=1 \\ n\neq m}}^{n=\infty} (T_n p + 1)\right] \Theta_0(t),$$

where p is a symbol of differentiation, for the initial conditions:

$$\Theta(x, y, z, t)|_{t=0} = \sum_{n=1}^{n=\infty} N_n(x, y, z);$$

$$\left.\frac{d\Theta(x, y, z, t)}{dt}\right|_{t=0} = -\sum_{n=1}^{n=\infty} \frac{N_n(x, y, z)}{T_n};$$

. . . .

$$\left.\frac{d^m\Theta(x, y, z, t)}{dt^m}\right|_{t=0} = (-1)^m \sum_{n=1}^{n=\infty} \frac{N_n(x, y, z)}{T_n^m}.$$

As can be seen, for the determination of the temperature at any point of the body being heated we must know in the general case not only the initial value of the temperature at this point, but also the initial temperature distribution; then it is possible to determine $N_n(x, y, z)$.

In the article we also consider the case of heating of a body on two sections of the surface by a medium having different temperatures on these sections.

LITERATURE CITED

1. V. S. Ermakov and G. I. Khutskii, Inzh.-Fiz. Zh., 14, No. 2 (1968).
2. A. Kh. Gorelik and M. A. Duel', Teploénergetika, No. 2 (1968).

Dep. 49-75, December 16, 1974.
Original article submitted October 23, 1972.

EXOTHERMIC RADICAL RECOMBINATION ON SURFACE OF SOLIDS

V. I. Gryadun and A. N. Gorban'

UDC 536.244+541.183

Recombination of free radicals, as is known [1], is usually an exothermic process. Solids on whose surface radicals (e.g. hydrogen, oxygen, nitrogen atoms) recombine are catalysts of this reaction and simultaneously absorb the energy released (of the order of a few electron-volts for each molecule formed).

The problem of heat distribution in a spherical solid on whose surface chemisorption and radical recombination take place is considered.

The thermal energy flux of radical recombination and the heat exchange with the medium are taken into account in the boundary conditions. The initial conditions assume that the temperatures of the sphere and its surroundings are the same.

The heat flux appearing in the boundary condition depends on the recombination coefficient and the concentration of chemisorbed radicals, which depends, in turn, on the surface temperature and the time. The Langmuir kinetic equation [2] is used to find the concentration of chemisorbed radicals on the surface. The coefficients of this equation, however, are functions of temperature, which makes it impossible to obtain an accurate expression for the kinetics of heating of the solid. In the case of low recombination coefficients on the given surface or low partial pressures of free radicals in the gas phase the solids are only slightly heated and the Langmuir coefficients can then be regarded as approximately constant during the experiment. It then becomes possible to find an expression for the heat flux and solve the posed problem, which in the considered case reduced to the known problem of heating of moist bodies in a medium with constant temperature, when evaporation occurs at the surface [3].

Values of the steady heating of various powdered crystal phosphors when hydrogen atoms recombine on them are given in [4]. These data agree well with the results of our calculation for the steady case.

LITERATURE CITED

1. V. A. Sokolov and A. N. Gorban', Luminescence and Absorption [in Russian], Nauka, Moscow (1969).
2. I. J. Langmuir, J. Amer. Chem. Soc., 38, 2217 (1916).
3. A. V. Lykov, Heat-Conduction Theory [in Russian], Vysshaya Shkola, Moscow (1967), p. 292.
4. V. G. Krongauz and B. P. Dmitriev, in: Luminescent Materials and Very Pure Substances [in Russian], No. 8, Stavropol (1973), p. 121.

Dep. 56-75, December 16, 1974.
Original article submitted July 17, 1974.

DISTRIBUTION OF TEMPERATURE, HEAT FLUX, AND ELECTROMAGNETIC FIELD IN A PLANE CONDUCTOR WITH TEMPERATURE-DEPENDENT CONDUCTIVITY (LOW FREQUENCIES)

R. S. Kuznetskii

UDC 538.56

In the determination of the distribution of temperature and electromagnetic field in current-heated conductors when the heat flux is high the local variation of the conductivity with temperature has to be taken into account. In a plane conductor $|x| \le 1$ of thickness $2a$ with cooled surfaces the established steady distributions of temperature and electromagnetic field are described by a system of nonlinear differential equations

$$t'' = -\sigma(u^2 + v^2),\quad u'' = -n^2\sigma v,\quad v'' = n^2\sigma u, \tag{1}$$

and $h = -in^{-2}e'$, with the following, for instance, symmetric boundary conditions: $u(1) = 1$, $v(1) = t(1) = u'(0) = v'(0) = t'(0) = 0$ $(0 \le x \le 1)$. Here x is the coordinate, t is the temperature relative to the surface temperature of the conductor, $\sigma = \sigma(t)$ is its conductivity (the dimensional conductivity when $t = 0$ is σ_0), $q = -t'$ is the heat flux density, $e \equiv u + iv$ is the complex amplitude of the electric field (the dimensional amplitude at the surface is e_0), and $h = n^{-2}(v' - iu')$ is the complex amplitude of the magnetic field; the corresponding dimensionless quantities are a, $(\sigma_0/\lambda)\cdot[(e_0 a)^2/(2-\delta_{s0})]$, σ_0, $\sigma_0 e_0^2 a/(2-\delta_{s0})$, e_0 and $\sigma_0 e_0 a$; $n \equiv a\sqrt{\sigma_0\mu\omega}$ is the frequency criterion; ω is the angular frequency of the current; λ is the thermal conductivity and μ is the absolute magnetic permeability; $\delta_{s0} \equiv \begin{cases}1, & n=0\\ 0, & n\ne 0\end{cases}$. From the functionals of the problem (1) we introduce the resistance $R = n^2/2\sigma_0 a \times v'/|e'|^2|_{x=1}$ and the internal inductance $L = (\mu a/2)\cdot(u'/|e'|^2)|_{x=1}$ per unit square surface of the conductor. Below we consider the behavior of the functions t(x), q(x), e(x), and h(x) and the functionals in the limiting case $n \ll 1$ (and $n = 0$), which is nontrivial when $\sigma' \ne 0$.

When $n = 0$ system (1) is integrated in quadratures, giving t(x) and q(x) (which we denote henceforth by the superscript 0), e, and h(x):

$$e = 1,\quad h = q = \sqrt{2\int_t^{t(0)} \sigma\, dt},\quad (1-x)\sqrt{2} = \int_0^t \frac{dt}{\sqrt{\int_t^{t(0)} \sigma\, dt}}; \tag{2}$$

the last equality determines the constant t(0) when $x = 0$. For nonzero $n \ll 1$ we obtain for $|e|$, $\varphi \equiv \arg e$, and $|h|$, $\psi \equiv \arg h$,

$$e \simeq 1 - n^2 y - i n^2 t^0,\quad |e| \simeq 1 - n^4\left[y - \frac{1}{2}(t^0)^2\right] \simeq 1,\quad \varphi \simeq -n^2 t^0 \le 0,\quad |\varphi| \ll 1; \tag{3}$$

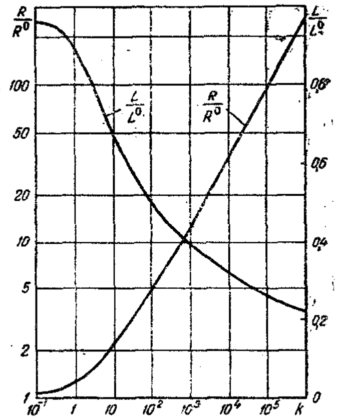

Fig. 1. Dimensionless resistance R/R^0 and internal inductance L/L^0 as function of nonlinearity criterion k at low frequencies (low-frequency criterion $n \le 1$).

$$h \simeq q^0 + n^4 J + in^2 y', \quad |h| \simeq q^0 + n^4 \left[J + \frac{(y')^2}{2q^0}\right] \simeq q^0, \quad \psi \simeq n^2 \frac{y'}{q^0} < 0, \quad |\psi| \ll 1, \tag{4}$$

where $\sigma^0 \equiv \sigma(t^0)$, $J(x) \equiv \int_0^x [(\sigma^0)'\vartheta - \sigma^0 y]dx$, $y(x) = \int_x^1 dx \int_0^x t^0\sigma^0 dx$ and $\vartheta(x)$ is the solution of equation

$$\vartheta'' + (\sigma^0)'\vartheta = \sigma^0 [2y - (t^0)^2], \quad \vartheta(1) = \vartheta'(0) = 0. \tag{5}$$

For R and L we have, respectively,

$$2\sigma_0 aR \simeq \frac{1}{q^0(1)} \left\{1 - \frac{n^4}{[q^0(1)]^2} [[y'(1)]^2 + q^0(1) J(1)]\right\} \simeq \frac{1}{q^0(1)}, \tag{6}$$

$$\frac{2}{\mu a} L \simeq - \frac{y'(1)}{[q^0(1)]^2} \left\{1 - \frac{n^4}{[q^0(1)]^2} [[y'(1)]^2 + 2q^0(1) J(1)]\right\} \simeq - \frac{y'(1)}{[q^0(1)]^2}. \tag{7}$$

All the obtained expressions are then made specific for the case of an inverse linear temperature dependence of the conductivity $\sigma(t) = (kt+1)^{-1}$ (to which Fig. 1 refers), where $k \equiv (e_0 a)^2 (\sigma_0/\lambda) \cdot [\alpha/(2 - \delta_{S0})]$ is the nonlinearity criterion of the problem (α is the temperature coefficient of the resistivity). In particular, when $k \gg 1$ we obtain the asymptotic relations

$$e \simeq 1, \quad t(0) \simeq \sqrt{\frac{2}{\pi k}},$$

$$q(1) \simeq \sqrt{\frac{\ln k}{k}} \simeq h(1), \tag{8}$$

$$R \simeq R^0 \sqrt{\frac{k}{\ln k}}, \quad L \simeq L^0 \frac{3}{\ln k},$$

where $R^0 = (2\sigma_0 a)^{-1}$, $L^0 = \mu a/6$ correspond to $k = n = 0$. The asymptotic formulas for the quantities of type (8) are valid irrespective of the value of n owing to their known self-similarity in the criterion n when $k \gg \max(1; n^4)$.

Dep. 3219-74, October 14, 1974.
Original article submitted March 15, 1974.

THERMAL FIELD OF AN INHOMOGENEOUS PLANE WITH CUTS

N. V. Pal'tsun and A. G. Gorban'

UDC 517.95

Let us determine the steady-state temperature field of a plane of unit thickness with a circular cutout of radius R. The plane is in contact with a circular plate of the same radius but of a different material. Along the line between the materials there are n cuts. In the plane and in the plate there are heat sources of strength g_1 and g_2, respectively, and at the edges of the cuts the temperature or the heat flux is specified.

We first determine the influence of the heat sources on the thermal field of this inhomogeneous plate, but without the cuts. We work from the equation for the temperature proposed by Prusov [1], which uses piecewise-holomorphic functions:

$$F_j^0(z) + F_j^0\left(\frac{R^2}{\bar{z}}\right) = T_j + i\eta_j, \tag{1}$$

$$F_j(z) + \left(\frac{R}{r}\right)^2 F_j\left(\frac{R^2}{\bar{z}}\right) = \frac{1}{iz} \cdot \frac{\partial}{\partial \theta} (T_j + i\eta_j), \tag{2}$$

$$F_j(z) - \left(\frac{R}{r}\right)^2 F_j\left(\frac{R^2}{\bar{z}}\right) = \frac{r}{z} \cdot \frac{\partial}{\partial r} (T_j + i\eta_j). \tag{3}$$

Here $F_j^0(z) = \int F_j(z)dz$ $(j = 1, 2)$; $F_j(z)$ are functions which are holomorphic everywhere in the regions $D^-(|z| > R)$, $D^+(|z| < R)$, except at the points $z = z_j$ and $z = R^2 \bar{z}_j^{-1}$, and $\eta(x, y)$ is some auxiliary harmonic function defined in the same region as $T(x, y)$. Assuming ideal thermal contact at the line L separating the media, we find the linear conjugate problem

$$(F_2 - F_1)^+ + (F_2 - F_1)^- = 0 \text{ on } L,$$
$$(F_2 + kF_1)^+ - (F_2 + kF_1)^- = 0 \text{ on } L, \qquad (4)$$

where $k = k_1/k_2$, and k_1 and k_2 are the thermal conductivities of the materials in regions D^- and D^+. After determining the piecewise holomorphic functions $F_1(z)$ and $F_2(z)$ from Eqs. (1), we find the temperature field of the inhomogeneous plane.

In solving this problem of a homogeneous plane with cuts we assume that boundary condition (4) holds in regions in which the media are in contact. Assuming this boundary condition, we write

$$F_2(z) - F_1(z) = F_3(z), \quad z \in D^+,$$
$$F_2(z) - F_1(z) = -F_3(z), \quad z \in D^-,$$
$$F_2(z) + kF_1(z) = F_4(z), \quad z \in D = D^- + D^+. \qquad (5)$$

Knowing that the functions $F_3(z)$ and $F_4(z)$ are piecewise-holomorphic over the entire plane of the complex variable, except at the cuts and at isolated singularities z_j and $R^2\bar{z}_j^{-1}$, we use (5) to find the functions $F_1(z)$ and $F_2(z)$ in terms of $F_3(z)$ and $F_4(z)$. The determination of the temperature field in the regions D^- and D^+ from (1)-(3) reduces to the problem of determining two functions, $F_3(z)$ and $F_4(z)$, which are piecewise-holomorphic in region D, as functions of the specified boundary conditions at the cuts. We assume that the values of the functions η_j and their normal derivatives at the sides of the cuts vanish.

This problem has been solved in closed form for Dirichlet and Neumann boundary conditions specified at the cuts.

The solution method is illustrated by an example.

LITERATURE CITED

1. I. A. Prusov, Certain Problems of Thermoelasticity [in Russian], Izd. BGU im. V. I. Lenina, Minsk (1972).

Dep. 53-75, December 16, 1974.
Original article submitted July 16, 1973.

ESTIMATE OF THE THAWING ZONES AND HEAT LOSS NEAR A PIPE BURIED IN FROZEN GROUND

B. A. Krasovitskii and B. L. Kriboshein

UDC 536.42:662.998

The planning of pipeline in frozen ground requires knowledge of the heat loss in the surrounding medium in order to correctly evaluate the productivity, and it is also necessary to know the size of the region in which the soil thaws in order to determine the conditions providing stability and strength of the pipe and its foundation. The general formulation of this problem can be solved only by numerical methods on powerful computers. For approximate estimates, which are worthwhile in the prediction stage, in the absence of surveying data, i.e., if the thermal and other physical characteristics of the soil are essentially unknown, a "cruder" model can be used. To find an approximate solution of this problem we use the following assumptions: a) The air temperature is constant, equal to the average annual temperature, and at a level below 0°C; b) in the thawed soil around a warm pipe the governing factor is the radial heat flux from the pipe, while in frozen soil near the air the governing factor is the vertical heat flux toward the surface with the air; c) the temperature distribution in the thawed and frozen zones if quasisteady, i.e., agrees with the steady-state solution for a given fixed position of the thawing boundary. Under these conditions the problem of determining the configuration of the thawing boundary reduces to the solution of a first-order partial differential equation. This equation, in turn, reduces to a system of two first-order ordinary differential equations. The latter are solved by means of standard programs on small computers. After the time evolution of the thawing region is found, it is a simple matter to evaluate the heat flux from the pipe to the soil. In addition, an equation was found for the heat flux into the soil over long periods of time.

The resulting solutions were used in calculations for specific pipelines. Comparison of the results of these calculations with the numerical solution of the problem shows that these solutions can be used to evaluate the heat flux from a pipeline in frozen soil and to determine the configuration and propagation of the thawing front.

Dep. 3216-74, October 29, 1974.
Original article submitted August 23, 1973.

APPROXIMATE METHOD OF CALCULATING FILTRATION PARAMETERS IN METALLOCERAMIC FILTERS

P. M. Zabridnii

The fundamental filtration parameters of metalloceramic filters are usually calculated by reference to formulas incorporating experimental coefficients. This method of calculation is troublesome and inaccurate.

The instantaneous delivery of a filter is equal to

$$\dot{W} = \frac{dW}{d\tau} = \frac{\Delta p F}{\mu(\zeta_1 + \zeta_2)}.$$

The volume of oil passing through the filter is

$$W = \frac{F\zeta_2}{x_0 r_0}\left(\sqrt{1 + \frac{2\Delta p x_0 r_0 \tau}{\mu\zeta_2^2}} - 1\right).$$

The resistance of the deposit formed on the surface of the filter barrier equals

$$\zeta_1 = \frac{r_0 x_0 W}{F},$$

while the resistance of the filter barrier itself is

$$\zeta_2 = \text{const}.$$

For a constant flow of oil the pressure drop in the filter, equal to

$$\Delta p = \mu V(\zeta_2 + V x_0 r_0 \tau),$$

should increase in accordance with a linear law until the permissible limit is reached, after which the filter should be cleaned or replaced.

If, however, the pressure drop is constant, the instantaneous delivery of the filter is

$$\dot{W} = \frac{F\Delta p}{\mu\zeta_2}\,\frac{1}{\sqrt{1 + \frac{2\Delta p}{\mu\zeta_2^2} x_0 r_0 \tau}},$$

i.e., it falls.

The resistance of the filter depends on the dimensions of the particles in the oil. The greatest resistance is offered by particles with dimensions slightly greater than the pores of the filter. The particles then stick in the pores and the number of open pores gradually diminishes.

Actually particles of varying size invariably fall into the oil; their distribution may be analyzed by means of a distribution function or histogram. In general we have the equation

$$\frac{dW}{d\tau}\left[\frac{r_0}{F_0}\int_0^\tau x_0 \dot{W} d\tau + \zeta_2(W, \tau)\right] = \frac{F\Delta p}{\mu}.$$

This equation may be solved approximately in the form of power series or by any of the well-known numerical methods; however, it is convenient to have an (even approximate) analytical expression for $W/\Delta p$.

The foregoing equation contains experimental quantities specified to an accuracy of 30%. We shall consider that $f = F\Delta p/\mu$ is specified to within 25-30%.

Using the method of experimental-coefficient variation developed by M. Ya. Brovman, we may regard any function $f_1, f_2, \ldots, f_n$ as taking values lying in the ranges of "equal probability" (25-30%) at the boundary of the region and at the initial instant of time. We should take account of the desirability of choosing the most "indefinite" quantities, since it is these which determine the expedient accuracy in the calculations.

From the latter equation we have

$$A\left[\frac{r_0}{F_0} Ax_0\left(\tau + \frac{k\tau^2}{2}\right) + \zeta_{20}(1 + Ab\tau + c\tau)\right] = f.$$

For $\tau = 0, A\zeta_{20} = f_0$.

Referring f to its exact solution f_0 and carrying out certain transformations, we obtain

$$\frac{f}{f_0} = 1 + \frac{A_1 bB\tau}{2\sqrt{1 + B\tau}} + c\tau + \frac{A_1 r_0 x_{00} \tau B}{2\sqrt{1 + B\tau}\, F_0 \zeta_{20}}\left(1 + \frac{k\tau}{2}\right).$$

The coefficients A_1 and B may be determined from the condition that f/f_0 should be specified at two particular instants of time.

If the quantity f/f_0 lies within the range of "equal probability" of the particular accuracy with which the quantities in the equation are specified, the linear approximation for $W(\tau)$ will constitute a satisfactory approximation.

Dep. 62-75, September 12, 1974.
Original article submitted March 22, 1974.